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Energy-efficient Daylighting Systems for Multi-story Buildings

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***Abstract*—Different fiber-based daylighting systems have been developed to improve indoor environments and reduce power consumption for electric lighting in office buildings. In this study, sunlight is captured through a concentrator and transmitted through optical fibers. Two approaches are investigated to improve the efficiency of fiber-based daylighting systems: the parabolic trough and linear Fresnel lens. Since it is difficult to achieve high concentration of light for optical fibers, a trough compound parabolic concentrator (CPC) is used upstream of the optical fibers to collect and concentrate light so that the maximum amount of captured collimated sunlight can be inserted into the optical fibers. The improvement in efficiency of these two approaches is evaluated by using simulation tools of LightTools, SolidWorks, and DIALux. The improvement is measured by calculating the average illuminance and illuminance uniformity in the interior. In addition, the illumination quality of the system is improved by combining of both daylight and LED (light-emitting diode) light. The results indicated that the efficiency achieved by either of the two approaches was better than that of the traditional lighting systems, and either approaches was found to save power consumption for lighting by maintaining an average illuminance of 500 lx, which is the minimum requirement for office buildings, at all times. Finally, the approach with the use of a parabolic trough resulted in more illuminance than that of the linear Fresnel lens.**

Keywords—solar concentrator; daylighting; nonimaging optics; optical fiber; energy efficiency

I. INTRODUCTION

Current directions toward green buildings possess energy efficiency, conversion of sources, renewable energy, and healthy environment. One of the principles of these buildings is to utilize natural light at all day to illuminate the interior. Daylighting has been recognized a major source to improve indoor environments. It is essential in buildings because it has various benefits such as light quality, energy efficiency, and psychological effect.

In South Korea, 43% of CO_2 emissions were produced from coal, 37% from oil, and 20% from Gas, and it was approached to 515.5 million tonnes of CO_2 in 2009 [1]. It has been estimated that 46% and 71% of total energy is consumed in buildings of South Korea and U.S., respectively [2]. Commercial energy consumption and industrial energy consumption in terms of electric lighting have been approached to 40–50% and 10–20%, respectively [3]. In sustainable buildings, energy saving is one of the main targets. Solar energy is one of the main sources of electricity production, and it can be used for daylighting. A reduction in energy consumption can be achieved by decreasing the use of artificial light in buildings. Daylight building is estimated to reduce lighting energy consumption by 50–80% [4]. In commercial buildings, electric lighting consumption can be reduced by 50–70% using daylight designing techniques with efficient artificial lighting controls [5]. The daylighting systems have been recognized to displace lighting energy consumption while providing very high quality illumination.

It has been estimated that natural light provides more light for less heat than artificial light. For example, it was monitored that a tungsten light source produces 5–14 times more heat than daylight [6]. To remove the heat, which is produced from each 100 watts of electric lighting, 20–50 watts are required [3]. In this case, we have to pay for both the cost to remove unwanted heat and the electric lighting. Approximately one watt of heat gain is attained from 100 lumens of illumination with sunlight [3].

Since light from the windows decreases very rapidly, and interior areas may not have sun exposure. As a result, the illumination is not consistent and some areas may remain dark. It has been estimated that about 70% of the room requires supplementary electric light, irrespective of the outdoor lighting conditions [7,8]. There should be some other source to provide sunlight for remaining areas to reduce electric lighting energy consumption. Therefore, we shall deliver daylight in those areas of the building through the proposed daylighting system. It guarantees to deliver sunlight deep into the core of the multi-floor buildings.

Numerous demonstrations have been presented to capture, guide, transport, and conversion of solar radiations such as solar concentrators, reflectors and lenses, light pipe and optical fiber, and solar thermal and solar photovoltaic, respectively [9]. Four main sunlight transport methods, namely, beam/lens system [10], mirror light pipe (MLP) [11], prismatic light pipe (PLP) [12], and optical fiber [7], have been examined. In the beam/lens method, the light through a collimating device is transported by arrangement of lenses and mirrors. In the MLP method, the sunlight is captured through lenses and reflectors and inserted into the light pipe and then light is transported and reflected from the inner surface of the pipe. PLP is made of prismatic dielectric surface in which light is trapped inside the light guide by total internal reflection. For daylighting, optical fiber is known as a good light transmission media. Light pipe is preferred to make the system cost-effective.

To follow the need of the market and energy codes, there should be simple methods in the daylighting system. Several concepts regarding daylighting systems have been demonstrated using optical fibers and light pipes [13-23], and most of the designs have a large number of tracking reflectors and lenses, and, thus, require an adequate area in which to be installed. In most of the cases, the building was illuminated by installing daylighting system separately for each floor. However, it increases the cost of the system. In order for daylighting system to be implemented on a large-scale, the cost of the system needs to be substantially reduced. It is better for the system to install the light collecting module on the roof of the building to use available free portion of the building. Therefore, we installed the sunlight collecting system on the roof of the building.

The remainder of the paper is organized in the following manner. Section II describes the architecture of the daylighting system and performance parameters. The detailed description of the light distribution in the interior is discussed in Section III. The complete study, involving the simulation, the implementation of the system, and the evaluation of the results are presented in Section IV. Finally, brief concluding remarks and future work are included in Section V.

II. Design of Daylighting Systems

Different techniques have been proposed in the literature to capture sunlight through reflectors and lenses. In most of the designs, a large number of concentrators were used to focus the light into optical fibers and light pipes. However, the system becomes costly when increasing the number of sunlight-capturing modules, and it is difficult to install sun-racking module to each concentrator. Usually, systems are preferred that have a minimum number of modules with an effective output while remaining cost effective. A daylighting system with a single concentrator is preferred instead of installing a system with many concentrators giving the same output as that of the single concentrator. In addition, the size of the sunlight-capturing module is selected according to the required range of illumination in the interior and the capacity of the light-transmission media.

In concentrated solar energy, the parabolic trough and linear Fresnel lens have gained popularity due to them being easily manageable, expandable, and widely available for rapid manufacturing compared to other concentrators (e.g., the parabolic dish) [24]. We preferred using these concentrators to produce highly concentrated sunlight. A large-scale daylighting system was needed to illuminate a large-scale building interior (e.g., a multi-floor building interior). Thus, a large number of optical fibers were required for light transmission. Each fiber should have the same intensity of light for the uniform distribution of light in different areas of the building. To this end, we propose two efficient approaches for the parabolic trough and the linear Fresnel lens. We achieve the uniform distribution of daylight at the capturing stage to increase the efficiency of the system.

A. *Parabolic trough*

The idea behind the system is to capture high-intensity sunlight and then focus it over the optical fibers. A parabolic trough captured sunlight and directed the light toward a parabolic reflector, which illuminated the optical fibers with collimated illumination. Here, the main issue was to achieve a very high concentration, which was very difficult. To solve the issue of high concentration, the trough CPC, which is a well-known, non-imaging optical element [25-28], was introduced upstream of the optical fibers. The trough CPC captured collimated light and then maximum sunlight was diverted into the optical fibers. For the parabolic trough, collimated light entered into the trough CPC through the parabolic trough and then light entered into the optical fibers, as shown in Fig. 1.

The optical fibers were exposed to maximum sunlight by arranging the fibers linearly. To allow for the maximum sunlight to enter into the optical fibers, collimated light was mandatory. The correct placement and measurements of reflectors to make collimated light was achieved by

$$F_1 = F_2 \tag{1}$$

where F_1 is the focal point of the parabolic trough and F_2 is the focal point of the concave reflector. All light on the surface of the optical fibers was accomplished by

$$H_{PR} = W_r \tag{2}$$

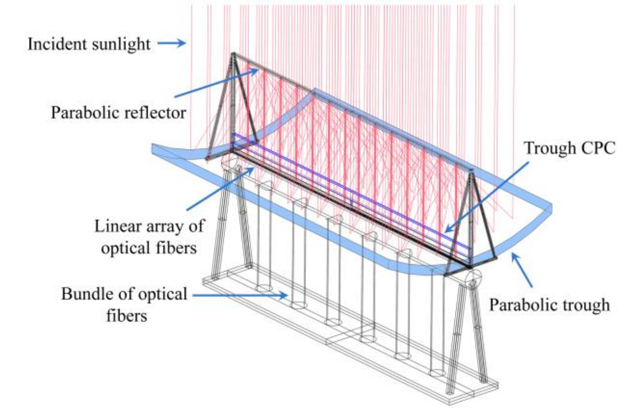

Figure 1. Physical layout of the system for the parabolic trough.

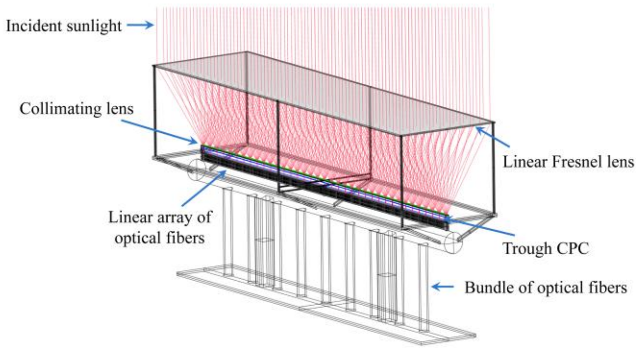

Figure 2. Physical layout of the system for the linear Fresnel lens.

where H_{PR} is the rectangular aperture height of the parabolic reflector and W_r is the width of the receiver. Here, the receiver corresponds to the linear array of the optical fibers. In our case, the value of W_r was 2 mm. Since the trough CPC was inserted before the optical fibers, the input of the trough CPC was collimated light. Therefore, collimated light can be easily concentrated into the optical fibers.

B. *Linear Fresnel lens*

The linear Fresnel lens was used to focus direct sunlight. The light went through a collimating lens and then the collimated light illuminated the optical fibers. It was difficult to achieve a very high concentration of light with the linear Fresnel lens. To demonstrate the system, perfect alignment, and a high concentration was necessary. Thus, we used the trough CPC upstream the optical fibers. As shown in Fig. 2, collimated light entered into the trough CPC through the linear Fresnel lens and then light entered into the optical fibers. Perfect alignment of the components was achieved to produce better results.

The Fresnel lens can be designed by varying the groove pitch, and it can also be designed by varying the groove depth with constant draft angle. We designed a linear Fresnel lens with a constant pitch. If we increased the width of the linear Fresnel lens, with its constant pitch, we would get lower efficiency due to blockage of the light. If we increased the rectangular aperture height of the parabolic trough, we would achieve higher efficiency. It can be concluded that a parabolic trough is more efficient than a linear Fresnel lens, while a linear Fresnel lens provides for an acceptable outcome at a low cost. The effective focal length (EFL) of the Fresnel lens can be calculated by

$$EFL = \frac{r}{n-1} \tag{3}$$

where r is the radius and n is the refractive index of the material. The material used was polymethylmethacrylate (PMMA), which had a refractive index of 1.494. Measurements of different parameters are mentioned in Table I. A combination of a linear Fresnel lens and a plano-concave lens (collimating lens) gave collimated light. The focal length of the collimating lens was calculated by [29]

$$NA = n \cdot sin\theta_{1/2} = \frac{D}{2f} \tag{4}$$

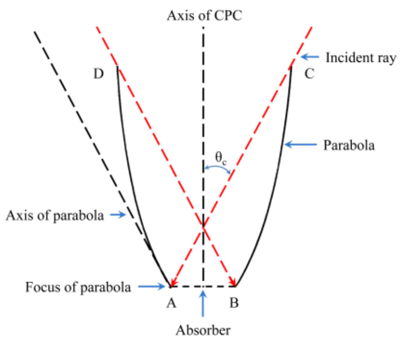

Figure 3. Layout of a simple CPC designed for a flat absorber.

where *NA* is the numerical aperture of the concentrator and *D* is the diameter of the collimating lens. All light on the fibers' surface was produced through the linear Fresnel lens by

$$W_p = W_r \tag{5}$$

where W_p is the width of the concave region of the plano-concave lens and W_r is the width of the receiver. Here, the receiver corresponds to the linear array of the optical fibers.

C. *Trough CPC*

In nonimaging optics, nonimaging concentrators (e.g., CPC) were introduced to achieve high concentration, especially for solar energy applications. Nonimaing concentrators are used to achieve high concentration for improving performance (efficiency) and the economics by reducing cost. Therefore, we used nonimaging concentrators for daylighting to provide high intensity illumination for saving energy in multi-floor buildings. It is very difficult to achieve a high concentration of light for optical fiber-based daylighting system using imaging concentrators. Practically, nonimaging concentrators have high efficiency in terms of concentration than imaging concentrators. Therefore, nonimaging concentrators have been used in solar thermal and concentrated photovoltaic systems.

CPC is a well-known nonimaging concentrator, which is used in many solar energy applications. We used CPC upstream of the optical fibers to achieve maximum concentration and insert maximum sunlight into optical fibers. Since we arranged optical fibers (absorber) in a linear array, we considered flat absorber for the CPC. The geometry of a simple CPC for maximizing the collection of incident radiation onto a flat absorber is shown in Fig. 3. The right branch (BC) is a segment of a parabola with its focus at point A. The left branch (AD) is a segment of a similar parabola corresponding to the image of (BC), obtained by reflection of BC through the symmetry axis (axis of CPC). This parabola has its focus at point B. The incident ray makes an angle θ_C with the axis of CPC. In two dimensions, we can say that 1) all rays from entrance aperture (DC) making an angle $\theta \leq \theta_C$ are reflected toward points on the flat absorber AB 2) all rays making an angle $\theta \geq \theta_C$ are rejected. The maximum geometric concentration is calculated by [27]

$$C_{max} = = \frac{n}{Sin\theta_c} \tag{6}$$

where *n* is the refractive index of the material. For *n=1*, we can find entrance aperture (*W*) by [27]

$$W = \frac{S}{Sin\theta_c} \tag{7}$$

where *S* is the width of the absorber. The height, *H*, of the parabola is [27]

$$H = \frac{S(1+1/Sin\theta_c)}{2Tan\theta_c} \tag{8}$$

Focal length, F, of the parabola is [27]

$$F = \frac{S}{2}\left(1 + Sin\,\theta_c\right) \tag{9}$$

To achieve thermodynamics limits for an ideal concentrator, the CPC must touch the absorber. A small gap is usually designed to reduce the heat problem. For small gap, the loss, L, is approximated by [27]

$$L \cong \frac{g}{\pi r} \tag{10}$$

where g is the gap and r is the radius of the absorber.

TABLE I. MEASUREMENTS OF DIFFERENT PARAMETERS

Title	**Value (mm)**
Rectangular aperture height of parabolic trough	1000
Rectangular aperture width of parabolic trough	570
Height of trough CPC	138.9
Width of parabolic reflector	15
Distance between both reflectors	456.7
Width of trough CPC	15
Height of trough CPC	89.75
Width of linear Fresnel lens	1000
Length of linear Fresnel lens	570
Width of collimating lens	15
Distance between both lenses	775

III. LIGHT TRANSMISSION AND DISTRIBUTION

We used optical fibers to deliver sunlight in the interior with small losses. Typically, an optical fiber is produced from different materials (e.g., glass and plastic). SOFs are known as good light-transmission media and have the best resistance to heating; however, SOFs are expensive. POFs have substantially higher attenuation coefficients than SOFs. As soon as the length of the POF is increased, the attenuation increases. Usually, POFs are preferred in daylighting systems due to their low cost, flexibility, strength, and acceptability for complex wiring in buildings.

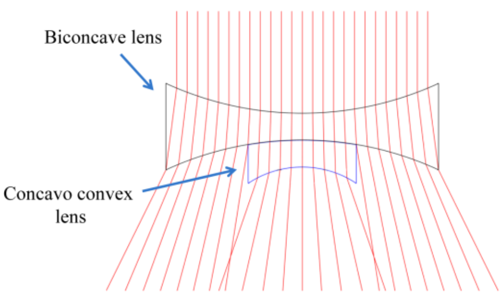

Figure 4. Layout of the diffusing structure.

In the proposed daylighting system, the SOFs had a length of 130 mm. After the SOFs, the POFs were used for most of the transmissions, and the length of the POFs varied because we placed fiber bundles in different areas. To reduce losses due to the air gap between the SOF and POF, index-matching gel was applied, as shown in Fig. 5(a). We used an acceptable size for the concentrators for a single array of optical fibers because the POF is sensitive to heat. In [7], we solved the heat problem through real implementation of POFs in daylighting applications.

Each fiber bundle had nineteen optical fibers. We used a combination of a biconcave lens and a concavo convex lens to distribute light uniformly, as illustrated in Fig. 4. The biconcave lens had a focal length of 13.2 mm and a radius of 12 mm. The concavo convex lens had a focal length of 11.3 mm and two radii of 12 mm and 3.6 mm. The focal length of the thin lens can be determined by the lens maker's formula, which can be written as [25]

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \tag{11}$$

where r_1 and r_2 are the two radii and n is the refractive index of the material. The lenses had a refractive index of 1.459.

IV. SIMULATION AND RESULTS

LightTools® was used to implement and simulate the proposed daylighting system. SolidWorks™ was used to design some 3D hardware modules and then these models were imported into LigfhtTools®. The room's interior was designed in DIALux™, and the complete optical system was imported into DIALux™ to show the room's interior under lighting simulation.

To achieve direct sunlight, it was presumed that both daylighting systems had sun-tracking devices to rotate the light-collecting modules toward the sun at all times of the day. In the simulation, the optical system was illuminated by collimated light, which was generated through a light source. The average illuminance values were calculated at different times of the day, as mentioned in Table. 2. For daylight, the luminous flux received on the surface was calculated to simulate the entire daylighting system by [26]

TABLE II. AVERAGE ILLUMINANCE AT DIFFERENT TIMES OF THE DAY AND THE CALCULATED FLUX FOR THE CONCENTRATOR.

Time	**Solar altitude (°)**	**Outdoor illuminance (lx)**	**Input flux (lm)**
12 PM	74	110000	62700
1 PM	65	105000	59850
2 PM	54	100000	57000
3 PM	43	80000	45600
4 PM	31	60000	34200
5 PM	19	40000	22800
6 PM	8	20000	11400

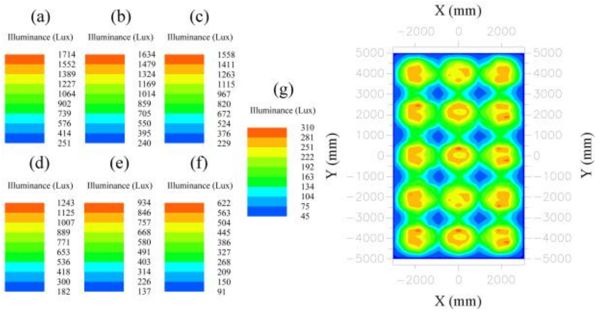

Figure 5. Daylight-illuminance distribution on the work plane for the linear Fresnel lens at (a) 12 PM, (b) 1 PM, (c) 2 PM, (d) 3 PM, (e) 4 PM, (f) 5 PM, and (g) 6 PM.

Figure 6. Room's interior view under lighting simulation.

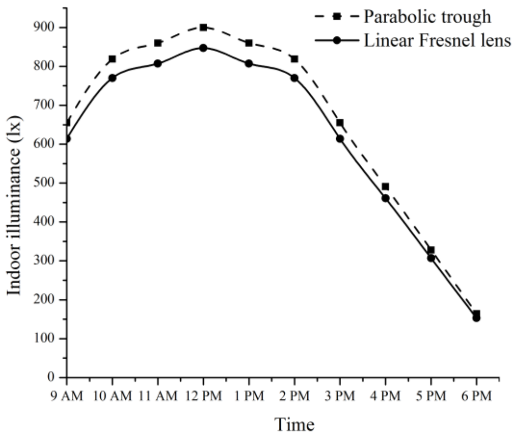

Figure 7. Average daylight-illuminance distribution on the work plane at different times of the day.

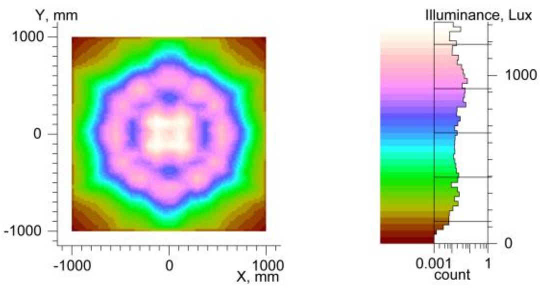

Figure 8. LEDs' illuminance distribution on the work plane.

$$E' = \frac{dF}{dS} \tag{12}$$

where dS is the (element of) surface area, dF is the luminous flux received on the (element) surface, and E' is the illuminance on the element surface. The illuminance distribution graph for the linear Fresnel lens is shown in Fig. 5. The interior view under lighting simulation is shown in Fig. 6. Average illuminance at different times of the day is shown in Fig. 7.

The illuminance uniformity of a given space (plane) is calculated by

$$Unifomity = \frac{E_{min}}{E_{avg}} \tag{13}$$

where E_{min} is the minimum level (value) of illuminance and E_{avg} is the average level (value) of illuminance. We achieved illuminance uniformity of 0.3 on the work plane for both approaches. Illuminance uniformity of 0.7 was attained on the floor for both approaches.

To deliver artificial light in cases where there is no daylight, we used an electric lighting-control system to maintain LED light. The basic principle of the hybrid system is that photocells sense the available light in the room, and the control algorithm maintains an average illuminance value in the interior. We used OSRAM™ LW-W5AM LEDs, which had a maximum optical efficiency of 130 lm/W [30]. To produce the LED light source, we arranged 26 LEDs in the parabolic reflector, which had a diameter of 130 mm and a depth of 192 mm. It illuminated an area of 4 m^2, and an average illuminance of 580 lx was achieved on the work plane, as shown in Fig. 8.

V. Conclusions

Two efficient approaches are proposed for a large-scale, optical fiber-based daylighting system. The light was focused through the parabolic trough and the linear Fresnel lens and then light was guided through the collimating device and the trough CPC. After that, POFs were used for light transmission, and a combination of a biconcave lens and a concavo convex lens was designed for uniform light distribution in the interior.

Previously, it was difficult to capture highly concentrated sunlight. We solved the high-concentration problem by using a collimating device and a CPC upstream of the optical fibers. We achieved the same intensity of light in each fiber to deliver uniform light in different areas of the building.

Optical-simulation tools were used to verify the efficiency of the system. We achieved uniform illumination at the capturing and distribution stages, and maximum sunlight was passed into optical fibers by minimizing losses. The results have shown that the efficiency of the system is better than that of traditional lighting systems. A comparative analysis was also carried out for the daylighting performance of the two approaches: the parabolic trough and linear Fresnel lens. It was found that the parabolic trough gave more illuminance than that of the linear Fresnel lens. Furthermore, we found that the hybrid system combining sunlight and LED light with electric lighting controls achieves the required illumination levels at all times, and it saves electric lighting power consumption by maintaining an average illuminance of 500 lx at all times, which is the minimum requirement for office buildings.

Both daylighting systems using the parabolic trough and linear Fresnel lens can be installed separately. As the proposed system is very suitable for large-scale building interiors, it can be installed for multi-floor buildings, where optical fibers can be distributed to each floor, as shown in Fig. 16. This study also shows that the daylighting system should be selected according to the interior size to save on hardware costs.

This study is the first to use a parabolic trough and linear Fresnel lens for the optical fiber-based daylighting systems. In the future, we will improve light uniformity in the interior of the building by distributing the light through the light guides, where light will be transmitted into the light guide through the bundle of optical fibers. Furthermore, we shall explore a daylighting system using heliostats for multi-floor buildings [19]. Finally, daylighting will help in reducing electric lighting power consumption and improving indoor environments.

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